LEVEL - VI

SINGLE ANSWER QUESTIONS

1. **Let** $f(x)=\lim_{n\to\infty}\dfrac{\left(x^2+2x+3+\sin\pi x\right)^n-1}{\left(x^2+2x+3+\sin\pi x\right)^n+1}$,

then

A) $f(x)$ is continuous and differentiable for all $x\in R$

B) $f(x)$ is continuous but not differentiable for all $x\in R$

C) $f(x)$ is discontinuous at infinite number of points.

D) $f(x)$ is discontinuous at finite number of points.

2. **Let**

$$f(x)=\lim_{h\to 0}\frac{\left(\sin(x+h)\right)^{\ln(x+h)}-(\sin x)^{\ln x}}{h}$$

then $f\left(\dfrac{\pi}{2}\right)$ **is**

A) equal to 0 B) equal to 1

C) $\ln\dfrac{\pi}{2}$ D) non existent

3. **Let** $g(x)=\begin{cases}\dfrac{x^2+x\tan x-x\tan 2x}{ax+\tan x-\tan 3x}; & x\neq 0\\ 0 ; & x=0\end{cases}$.

If $g'(0)$ **exists and is equal to non zero value b, then** $\dfrac{b}{a}$ **is equal to**

A) $\dfrac{7}{13}$ B) $\dfrac{7}{26}$ C) $\dfrac{7}{52}$ D) $\dfrac{5}{52}$

4. **If** $\lim_{n\to\infty}\dfrac{n.3^n}{n(x-2)^n+n.3^{n+1}-3^n}=\dfrac{1}{3}$ **where** $n\in N$, **then the number of interger(s) in the range 'x' is**

A) 3 B) 4 C) 5 D) infinite

5. **Let**

$$f(x)=\frac{e^{\tan x}-e^x+\ln(\sec x+\tan x)-x}{\tan x-x}$$

be a continuous function at x = 0. The value of $f(0)$ **equals**

A) $\dfrac{1}{2}$ B) $\dfrac{2}{3}$ C) $\dfrac{3}{2}$ D) 2

6. $f(x)=\begin{cases}\dfrac{\cos^{-1}\left(1-\{x\}^2\right)\sin^{-1}\left(1-\{x\}\right)}{\sqrt{2}\left(\{x\}-\{x\}^3\right)}, & for\ x\neq 0\\ \dfrac{\pi}{2}, & for\ x=0\end{cases}$

where $\{.\}$ **denotes fractional part of** x**, then, at** $x=0$, $f(x)$**, is**

a) continuous b) only left continuous

c) only right continuous

d) neither left continuous nor right continuous

7. **If f is a periodic function with period T &** $[.]$ **denotes GIF, then,**

$$\lim_{n\to\infty}\frac{\left[f(x+T)^{f(x+T)}\right]+\left[2^2 f(x+2T)^{f(x+2T)}\right]+\left[3^2 f(x+3T)^{f(x+3T)}\right]+.....+\left[n^2 f(x+nT)^{f(x+nT)}\right]}{n^3}$$

a) $\dfrac{\left(f(x)\right)^{f(x)}}{2}$ b) $\dfrac{\left(f(x)\right)^{f(x)}}{4}$

c) $\dfrac{2\left(f(x)\right)^{f(x)}}{3}$ d) $\dfrac{\left(f(x)\right)^{f(x)}}{3}$

8. **If f(x) is continuous function** $\forall x\in R$ **and the range of** $f(x)$ **is** $\left(2,\sqrt{26}\right)$ **and**

$g(x)=\left[\dfrac{f(x)}{c}\right]$ **is continuous** $\forall x\in R$,

then the least positive integral value of c is (where [.] denotes the greatest integer function)

(A) 2 (B) 3 (C) 5 (D) 6

9. **The function $f(x)$ defined by**

$$f(x)=\begin{cases}\log_{(4x-3)}\left(x^2-2x+5\right), & \frac{3}{4}<x<1 \text{ and } x>1\\ 4, & x=1\end{cases}$$

A) is continuous at x = 1

B) is discontinuous at x = 1 since $f\left(1^+\right)$ does not exist though $f\left(1^-\right)$ exists.

C) is discontinuous at x = 1 since $f\left(1^-\right)$ does not exist though $f\left(1^+\right)$ exists.

D) is discontinuous at x = 1 since neither $f\left(1^+\right)$ nor $f\left(1^-\right)$ exists.

10. $f(x)=\max\left\{\frac{x}{n},\left|\sin \pi x\right|, n\in N\right\}$ **has maximum points of non-differentiabilityu for $x\in(0,4)$, then**

A) maximum value of n in more than 4.5
B) least value of n is more that 3.5
C) maximum value of n is less than 4.5
D) least value of n is less than 3.5

11. **Let n be a positive integer, for n=1,2,3,...., n let S_K denotes the area of ΔAOB_k such that $\angle AOB_k=\frac{k\pi}{2n}$, OA=1, $OB_k=k$. The value of $\lim\limits_{n\to\infty}\frac{1}{n^2}\sum_{n^2}^{\infty}S_k$ is**

A) $\frac{2}{\pi^2}$ B) $\frac{4}{\pi^2}$ C) $\frac{8}{\pi^2}$ D) $\frac{1}{2\pi^2}$

12. **The value of $\sum_{n=2}^{\infty}\ln\left(1-\frac{1}{n^2}\right)$ equals**

A) –ln3 B) 0 C) –ln 2 D) –ln 5

13. **Let $f:R\to R$ be a continuous into function satisfying $f(x)+f(-x)=0,\forall\ x\in R$. If $f(-3)=2$ and $f(5)=4$ in $[-5,5]$, then the equation $f(x)=0$ has**

A) exactly three real roots
B) exactly two real roots
C) atleast five real roots
D) atleast three real roots

14. **Let x_n be defined as $\left(1+\frac{1}{n}\right)^{n+x_n}=e$, then $\lim\limits_{n\to\infty}x_n$ equals**

A) 1 B) $\frac{1}{2}$ C) $\frac{1}{e}$ D) 0

15. **Let $f(x)$ be continuous for all $x\in R$ except at x = 0 and $f'(x)<0\ \forall\ x\in(-\infty,0)$; $f'(x)>0\ \forall\ x\in(0,\infty)$. Let $\lim\limits_{x\to0^+}f(x)=3,\ \lim\limits_{x\to0^-}f(x)=4,\ f(0)=5$. Then the image of $(0,1)$ about the line $y\lim\limits_{x\to0}f\left(\cos^3x-\cos^2x\right)=x\lim\limits_{x\to0}f\left(\sin^2x-\sin^3x\right)$ is**

A) $\left(\frac{12}{25},\frac{-9}{25}\right)$ B) $\left(\frac{11}{25},\frac{-9}{25}\right)$

C) $\left(\frac{16}{25},\frac{-8}{25}\right)$ D) $\left(\frac{24}{25},\frac{-7}{25}\right)$

MULTI ANSWER QUESTIONS

16. **Given $f(x)=\sum_{r=1}^{n}\left(x^r+x^{-r}\right)^2$; $x\neq\pm1$ and**

$$g(x)=\begin{cases}\lim\limits_{n\to\infty}\left(\left(f(x)-2n\right)x^{-2n-2}\left(1-x^2\right)\right) & \text{for } x\neq\pm1\\ -1, \text{ for } x=\pm1\end{cases}$$

then $g(x)$

A) is discontinuous at x = –1
B) is continuous at x = 2
C) has a removable discontinuity at x = 1
D) has an irremovable discontinuity at x = 1

17. **If $a,b,c\in R^+$ then $\lim\limits_{n\to\infty}\sum_{k=1}^{n}\frac{n}{(k+an)(k+bn)}$ is equal to**

A) $\frac{1}{a-b}\ln\frac{b(b+1)}{a(a+1)}$ if $a\neq b$

B) $\frac{1}{a-b}\ln\frac{a(b+1)}{b(a+1)}$ if $a\neq b$

C) non existent if a = b

D) equals $\frac{1}{a(1+a)}$ if a = b

18. f(x) = min {1, cos x, 1-sin x}, $-\pi \leq x \leq$ p, then

(A) f(x) is not differentiable at 0
(B) f(x) is differentiable at $\pi/2$
(C) f(x) has local maxima at 0
(D) f(x) local maximum at x= $\pi/2$

19. If f(x) = min (tan x, cot x), then

(A) f(x) is discontinuous at $x = 0, \frac{\pi}{4}, \frac{5\pi}{4}$

(B) f(x) is continuous at $x = 0, \frac{\pi}{2}, \frac{3\pi}{2}$

(C) $\int_0^{\pi/2} f(x)dx = 2\ln\sqrt{2}$

(D) f(x) is periodic with period π

20. The function f(x) = $\left|\left|e^x - 1\right| - 1\right|$ is

(A) continuous for all x
(B) differentiable for all x
(C) not continuous at x = 0, ln 2
(D) not differentiable at $x = \ln 2$

21. Given a real valued function *f* such that

$$f(x) = \begin{cases} \frac{\tan^2\{x\}}{\left(x^2 - [x]^2\right)} & for\ x > 0 \\ 1 & for\ x = 0 \\ \sqrt{\{x\}\cot\{x\}} & for\ x < 0 \end{cases}$$ **, where [x] is the integral parrt and {x} is the fractional part of x, then**

A) $\lim_{x\to 0^+} f(x) = 1$

B) $\lim_{x\to 0^-} f(x) = \cot 1$

C) $\cot^{-1}\left(\lim_{x\to 0^-} f(x)\right)^2 = 1$

D) $\tan^{-1}\left(\lim_{x\to 0^+} f(x)\right) = \frac{\pi}{4}$

22. If $f(x) = \text{sgn}\left(x^2 - ax + 1\right)$ has maximum number of points of discontinuity, then

A) $a \in (2, \infty)$ B) $a \in (-\infty, 2)$

C) $a \in (-2, 2)$ D) $a \in [-2, 2]$

23. If $f(x) = \begin{cases} x^2\left(\text{sgn}[x] + \{x\}\right), & 0 \leq x < 2 \\ \sin x + |x - 3|, & 2 \leq x < 4 \end{cases}$**, where [] and {} represent the greatest integer and the fractional part function, respectivley.**

A) $f(x)$ is differentiable at x = 1

B) $f(x)$ is continuous but non-differentiable at x = 1

C) $f(x)$ is non-differentiable at x = 2

D) $f(x)$ is discontinuous at x = 2.

ASSERTION & REASON QUESTIONS

24. Which of the following statement(s) is/are TRUE?

Statement A: If function $y = f(x)$ is continuous at x = c such that $f(c) \neq 0$ then $f(x)f(c) > 0\ \forall x \in (c-h, c+h)$ where h is sufficiently small positive quantity.

Statement B:

$$\lim_{n\to\infty}\frac{1}{n}\ln\left[\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)......\left(1+\frac{n}{n}\right)\right] = 1 + 2\ln 2$$

Statement C: Let f be a continuous and non-negative function defined on [a, b]. if $\int_a^b f(x)dx = 0$ then $f(x) = 0\ \forall\ x \in [a,b]$.

Statement D: Let f be a continuous function defined on [a, b] such that $\int_a^b f(x)dx = 0$, then there exists atleast one $c \in (a,b)$ for which $f(c) = 0$.

COMPREHENSION QUESTIONS

PASSAGE ::1

$$f(x)=\begin{cases} x+a & x<0 \\ |x-1| & x\geq 0 \end{cases}$$

$$g(x)=\begin{cases} x+1 & x<0 \\ (x-1)^2+b & x\geq 0 \end{cases}$$

Where a and b are non – negative real numbers

25. **The value of a, if $(gof)(x)$ is continuous for all real x , is**
(A) – 1 (B) 0 (C) 1 (D) 2

26. **The value of b , if $(gof)(x)$ is continuous for all real x , is**
(A) – 1 (B) 0 (C) 1 (D) 2

27. **For these values of a and b , $(gof)(x)$ for all x $\in(-1,1)$ *is***
(A) Even (B) odd
(C) neither even nor odd
(D) symmetrical about x – axis

PASSAGE ::2

Two functions $f(x)\,\&\,g(x)$ are defined as,

$$f(x)=\begin{cases} [h(x)]-\left\{\dfrac{h(x)}{2}\right\}, & \text{for } x\in \text{domain of h} \\ 0 & ,\text{for } x\notin \text{domain of h} \end{cases}$$

$$g(x)=\begin{cases} sgn\ h(x), & \text{for } x\in \text{domain of h} \\ 0 & ,\text{for } x\notin \text{domain of h} \end{cases}$$

Where,

$$h(x)=\frac{1}{\sqrt{b-a}}.\frac{\sqrt{\dfrac{b-a}{a}}\sin 2x}{\sqrt{1+\left(\sqrt{\dfrac{b-a}{a}}\sin x\right)^2}}.\sqrt{a+b\tan^2 x}$$

for $b>a>0$ & $[.]$ denotes G.I.F & $\{.\}$ denotes fractional part of x. Now answer the following.

28. **For $h(x)$ which one of the following is true.**

a) $h(x)$ is continuous at $x=0$ and $x=\frac{\pi}{2}$

b) $h(x)$ is continuous at $x=0$ but discontinuous at $x=\frac{\pi}{2}$

c) $h(x)$ is discontinuous at $x=0$ but continuous at $x=\frac{\pi}{2}$

d) $h(x)$ is discontinuous at $x=0$ & $x=\frac{\pi}{2}$

29. **For $f(x)$, which of the following is true**

a) $f(x)$ is continuous at $x=0$ and $x=\frac{\pi}{2}$

b) $f(x)$ is continuous at $x=0$ but discontinuous at $x=\frac{\pi}{2}$

c) $f(x)$ is discontinuous at $x=0$ but continuous at $x=\frac{\pi}{2}$

d) $f(x)$ is discontinuous at $x=0$ & $x=\frac{\pi}{2}$

30. $g(\pi/4)+g(2\pi/3)+g(0)+g(\pi/2)=$
a) 0 b) 1
c) -1 d) 2

PASSAGE ::3

Let $f(x)=\begin{cases} x+2, & 0\leq x<2 \\ 6-x, & x\geq 2 \end{cases}$,

$$g(x)=\begin{cases} 1+\tan x, & 0\leq x<\dfrac{\pi}{4} \\ 3-\cot x, & \dfrac{\pi}{4}\leq 4<\pi \end{cases}$$

31. $f\left(g\left(x\right)\right)$ **is**

A) discontinuous at $x=\frac{\pi}{4}$

B) differentiable at $x=\frac{\pi}{4}$

C) continuous but non-differentiable at $x=\frac{\pi}{4}$

D) differentiable at $x=\frac{\pi}{4}$, but derivative is not continuous.

32. The number of points on non-differentiability of $h(x)=\left|f\left(g\left(x\right)\right)\right|$ is

A) 1 B) 2 C) 3 D) 4

33. The range of $h(x)=f\left(g\left(x\right)\right)$ is

A) $(-\infty,\infty)$ B) $(4,\infty)$

C) $(-\infty,4)$ D) [-4, 5]

PASSAGE ::4

Let $f\left(x\right)=2+\left|x-1\right|$ and

$g\left(x\right)=\min\left(f\left(t\right)\right)$ where $x\le t\le x^2+x+1$

then answer the following:

34. Number of points of discontinuity of $g\left(x\right)$ is

A) 1 B) 2 C) 3 D) 0

35. Number of points where the function $g\left(x\right)$ is not differentiable is

A) 1 B) 2 C) 3 D) 0

36. Range of $g\left(x\right)$ is

A) $[2,\infty)$ B) $[2,2)$ C) $[0,2]$ D) $(-\infty,\infty)$

PASSAGES ::5

Let A be a $n\times n$ matrix given by

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{bmatrix}$$ **such that**

each horizontal row is an arithmetic progression and each vertical column is a geometrical progression. It is known that each column in geometric progression have the same common ratio. Given that $a_{24}=1$, $a_{42}=\frac{1}{8}$ and $a_{43}=\frac{3}{16}$.

37. Let $S_n=\sum_{j=1}^{n}a_{4j}$, then $\lim_{n\to\infty}\frac{S_n}{n^2}$ is equal to

A) $\frac{1}{4}$ B) $\frac{1}{8}$

C) $\frac{1}{16}$ D) $\frac{1}{32}$

38. Let d_i be the common difference of the elements in i^th^ row, then $\sum_{i=1}^{n}d_i$

A) n B) $\frac{1}{2}-\frac{1}{2^{n+1}}$

C) $1-\frac{1}{2^n}$ D) $\frac{n+1}{2^n}$

39. The value of $\lim_{n\to\infty}\sum_{i=1}^{n}a_{ii}$ is equal to

A) $\frac{1}{4}$ B) $\frac{1}{2}$

C) 1 D) 2

PASSAGE: 6

If $f\left(x\right)=x^2-2\left|x\right|$

$g(x)=\text{minimum}\left\{f\left(t\right):-2\le t\le x,\ -2\le x<0\right\}$

$\quad\quad \text{minimum}\left\{f\left(t\right):0\le t\le x,\quad 0\le x\le 3\right\}$

then

40. The function $y=\left|f\left(x\right)\right|$ is differentiable for

A) $x\in R$ B) $x\in R-\{0\}$

C) $x\in R-\{0,2\}$ D) (-2, 2)

41. The function $g\left(x\right)$ is differentiable for

A) $x\in[-2,3]$

B) $x\in[-2,3]-\{-1,0,1\}$

C) $x\in[-2,3]-\{0,1\}$

D) $x\in[-2,3]-\{-1,0,2\}$

42. $\int_0^1 fog(x)\,dx$ **is**

A) 0 B) $\frac{33}{2}$ C) 21 D) 7 / 3

PASSAGE: 7

Suppose f, g and h be three real valued function defined on R.

Let $f(x)=2x+|x|$, $g(x)=\frac{1}{3}(2x-|x|)$ **and** $h(x)=f(g(x))$

43. The range of the function $k(x)=1+\frac{1}{\pi}\left(\cos^{-1}(h(x))+\cot^{-1}(h(x))\right)$ **is equal to**

A) $\left[\frac{1}{4},\frac{7}{4}\right]$ B) $\left[\frac{5}{4},\frac{11}{4}\right]$ C) $\left[\frac{1}{4},\frac{5}{4}\right]$ D) $\left[\frac{7}{4},\frac{11}{4}\right]$

44. The domain of definition of the function $l(x)=\sin^{-1}(f(x)-g(x))$ **is equal to**

A) $\left[\frac{3}{8},\infty\right)$ B) $(-\infty,1]$

C) $[-1,1]$ D) $\left(-\infty,\frac{3}{8}\right]$

45. The function $T(x)=f(g(f(x)))+g(f(g(x)))$ **is**

A) continuous and diferentiable in $(-\infty,\infty)$
B) continuous but not derivable $\forall x\in R$
C) neither continuous nor derivable $\forall x\in R$
D) an odd function

PASSAGES:: 8

Consider the function $f:R\to R$**, defined**

as $f(x)=\begin{cases} x^2-x+3, & x\in(-\infty,3)\cap Q \\ x+a, & x\in(-\infty,2)-Q \\ 2^x+1, & x\in(2,3)-Q \\ 9\tan\left(\frac{\pi x}{12}\right), & x\in[3,6) \end{cases}$

46. If $f(x)$ **is continuous at x = 2 then the vlaue of a is**

A) 1 B) 2 C) 3
D) indeterminate

47. The function $f(x)$ **at x = 3**

A) has non-removable discontinuity
B) has removable discontinuity
C) is differentiable
D) is continuous but not differentiable

48. $f'(4)$ **is equal to**

A) π B) 3π C) $\frac{3\pi}{2}$ D) $\frac{3\pi}{16}$

MATRIX MATCHING QUESTIONS

49. Match the following :

Column I (Functions)

A) $f(x)=|x|$

B) $f(x)=x^n|x|, n\in N$

C) $f(x)=\begin{cases} x\ln|\sin x|, & x\neq 0 \\ 0, & x=0 \end{cases}$

D) $f(x)=\begin{cases} x\,e^{1/x}, & x\neq 0 \\ 0, & x=0 \end{cases}$

Column II (Properties)

P) Continuous at $x=0$
Q) Discontinuous at $x=0$
R) Differentiable at $x=0$
S) Non- differentiable at $x=0$

50. Column I

A) $f(x)=\begin{cases} \frac{1}{|x|} & for\ |x|\geq 1 \\ ax^2+b & for\ |x|<1 \end{cases}$ is differentiable every where and $|k|=a+b$, then value of k is

B) If $f(x)=\operatorname{sgn}(x^2-ax+1)$ has exactly one point of discontinuity, then the value of a can be

C) $f(x)=[2+3|n|\sin x], n\in N, x\in(0,\pi)$ has exactly 11 points of discontinuity, then the value of n is

D) $f(x)=||x|-2+a|$ has exactly three points of non-differentiability, then the value of a is

Column II

P) 2 ; Q) –2 ; R) 1 ; S) –1

51. **Let $f(x)$ be a real valued function defined by $f(x)=x^2-2|x|$ and**

$$g(x)=\begin{cases}\text{minimum}\{f(t):-2\le t\le x\}, & x\in[-2,0)\\ \text{maximum}\{f(t):0\le t\le x\}, & x\in[0,3]\end{cases}$$

Column I

A) $g(x)$ is not continuous at x equal to

B) $g(x)$ is not derivable at x equal to

C) Number of integral critical points of $g(x)$ is equal to

D) Absolute maximum valueof $g(x)$ is equal to

Column II

P) –2
Q) 0
R) 1
S) 2
T) 3

52. **Match the following :**

Column I

A)

$$f(x)\begin{cases}\dfrac{\left(\dfrac{\pi}{2}-\sin^{-1}\left(1-\{x\}^2\right)\right)\left(\sin^{-1}\left(1-\{x\}\right)\right)}{\sqrt{2}\left(\{x\}-\{x\}^3\right)} & x>0\\ k & x=0\\ \dfrac{A\sin^{-1}\left(1-\{x\}\right)\cos^{-1}\left(1-\{x\}\right)}{\sqrt{2\{x\}}\left(1-\{x\}\right)} & x<0\end{cases}$$

is continuous at x = 0 then the value of $\sin^2 k+\cos^2\left(\dfrac{A\pi}{\sqrt{2}}\right)$ is

B) $f(x)=\left[2+5|n|\sin x\right]n\in Z$ has exactly 19 points of non differentiability in $x\in(0,\pi)$ then possible values of n are

C) If $f(x)=\begin{cases}x.\dfrac{(3/4)^{1/x}-(3/4)^{-1/x}}{(3/4)^{1/x}-(3/4)^{-1/x}} & x\ne 0\\ 0 & x=0\end{cases}$

if $P=f'(0^-)-f'(0^+)$ then

$$\lim_{x\to P^-}\frac{\left(\exp(x+2)\ln 4\right)^{\frac{[x+1]}{4}}-16}{4^x-16}$$ is $\le$

Column II

P) 2
Q) –2
R) 3
S) –3

INTEGER QUESTIONS

53. **If** $\lim\limits_{x\to 0}\dfrac{1-\cos\left(1-\cos\dfrac{x}{2}\right)}{2^m x^n}$ **is equal to the left hand derivative of** $e^{-|x|}$ **at x = 0, then find the value of** $\left(n^2+m\right)$**.**

54. **If** $f(x)=\begin{cases}\text{sgn}(x-2)\times[\log_e x], & 1\le x\le 3\\ \{x^2\}, & 3<x\le 3.5\end{cases}$**, where $[\bullet]$ denotes the greatest integer function and $\{\bullet\}$ represents the fractional part function, then the number of points of discontinuity is................**

55. **Let** $\lim\limits_{x\to 1}\left[\dfrac{x^a-ax+a-1}{(x-1)^2}\right]=f(a)$**. Then the value of** $f(4)$ **is ....**

56. **If f and g are two differentiable functions with $g'(a)=2,\ g(a)=b,$ such that fog = identity function, then $2f'(b)$ is equal to ....**

57. **If** $f(x), g(x)$ **be twice differentiable function on [0,2] satisfying** $f''(x) = g''(x), f'(1) = 2, g'(1) = 4$ **and** $f(2) = 3, g(2) = 9,$ **then –f(x) + g(x) at x = 2 equals …………..**

58. **Let S denote sum of the series** $\frac{3}{2^3} + \frac{4}{2^4.3} + \frac{5}{2^6.3} + \frac{6}{2^7.5} + ...\infty$. **Then the vlaue of** S^{-1} **is**

59. **The vlaue of**

$$\lim_{n\to\infty}\left(\frac{1}{2n+1} + \frac{1}{2n+3} + \frac{1}{2n+5} + ... + \frac{1}{4n-1}\right)$$ **is**

$\frac{a}{b}\ln c$ **where** $a, b, c \in N$. **Then the least value of** $a + b + c$ **is**

60. **Let** $I_n = \int_{-1}^{1} |x|\left(1 + x + \frac{x^2}{2} + \frac{x^3}{3} + ... + \frac{x^{2n}}{2n}\right) dx$.

If $\lim_{n\to\infty} I_n$ **can be expressed as rational** $\frac{p}{q}$ **in the lowest form, then the value of pq is**

61. **The value of**

$$\left[\lim_{n\to\infty}\left(2\times 3^2 \times 2^3 \times 3^4.....2^{n-1} \times 3^n\right)^{\frac{1}{(n^2+1)}}\right]^4$$ **is**

______

62. **The integral value of** ***n*** **for which** $\lim_{x\to 0}\left\{\left[\cos^2 x - \cos x - e^x \cos x + e^x - \left(x^3/2\right)\right]/x^n\right\}$ **is finite and non-zero is _______**

63. **If** $L = \lim_{x\to 0} \frac{\sin x^4 - x^4 \cos x^4 + x^{20}}{x^4\left(e^{2x^4} - 1 - 2x^4\right)}$, **then the value of 1/L is __________**

64. **If** $\lim_{x\to 0} \sin\left(\frac{\pi\left(1 - \cos^m x\right)}{x^n}\right)$ **exists, where** $m, n \in N$, **then the sum of all possible values of** ***n*** **is __________.**

65. **If** $\lim_{x\to\infty}\left(e^{2x} + e^x + x\right)^{\frac{1}{x}} = e^a$, **then the value of** a **is**

66. **If** a **and** b **are the numbers of points of non differentiability of** $f(x) = \left[\sin^{-1} x\right]$ **and** $f(x) = \left[\frac{2}{1+x^2}\right], x \ge 0$, **(where [.] represents greatest integer function) respectively, then the value of** $a + b$ **is _____.**

67. **The least integral value of** a **for which the funciton** $f(x) = \left[(x-2)^3 / a\right]\sin(x-2) + a\cos(x-2)$, **where [.] denotes teh greatest integer function, is continuous in [0,2] is _____**

68. **If** a **is the number of points of continuity of** $f(x) = \begin{cases} x - 1, & x \text{ is rational} \\ x^2 - x - 2, & x \text{ is irrational} \end{cases}$, b **is the number of points of discontinuity of** $f(x) = \text{sgn}\left(x^3 - 3x + 1\right)$, ***c*** **is the number of points of non-differentiability of** $f(x) = (\log x)\left|x^2 - 4x + 3\right| + 2(x-2)^{1/3}$, **and** ***d*** **is the number of points where the graph of** $f(x) = (\log x)\left|x^2 - 4x + 3\right| + 2(x-2)^{1/3}$ **has a sharp turn then the value of** $a + b + c + d$ **is _____.**

69. **The number of points of discontinuity of** $f(x) = \lim_{n\to\infty}\left[\left(x^{2n} - 1\right)/\left(x^{2n} + 1\right)\right]$ **is ______**

70. **Given the continuous function** $f(x) = \begin{cases} 2x + 3 & x \le 1 \\ -x^2 + 6 & x > 1 \end{cases}$, **then numner of points where** $\left|f(|x|)\right|$ **is non-differentiable is _________.**

KEY - LEVEL - VI

SINGLE ANSWER QUESTIONS

1. A	2. A	3. C	4. C
5. C	6. C	7. D	8. D
9. D	10. B	11. A	12. C
13. D	14. B	15. D	

MULTI ANSWER QUESTIONS

16. A,B,D	17. B, D
18. A, C	19. C, D
20. A, D	21. A, B, C, D
22. A, B	23. A, C, D
24. A,C, D	

COMPREHENSION QUESTIONS

25. C	26. B	27. A
28. B	29. D	30. A
31. C	32. B	33. C
34. C	35. C	36. A
37. D	38. C	39. D
40. B	41. C	42. A
43. B	44. D	45. B
46-C,	47-D	48. B

MATRIX MATCHING QUESTIONS

49.(A – p,s) (B – p,r) (C – p,s) (D – q,s)
50. A-R, S; B-P, Q; C-P, Q; D-P, R
51. A-Q, B-Q, S, C-T, D-T
52. A-P, B- P, Q, C-Q, R

INTEGER QUESTIONS

53. 9	54. 4	55. 6	56. 1
57. 6	58. 2	59. 5	60. 6
61.6	62. 4	63. 6	64. 3
65. 2	66. 5	67. 9	68. 8
69. 2	70. 5		

HINTS-LEVEL - VI

SINGLE ANSWER QUESTIONS

1. $x^2+2x+3+\sin\pi x=(x+1)^2+2+\sin\pi x>1$

$\therefore\ f(x)=1\ \forall\ x\in R$

2. If $g(x)=(\sin x)^{\ln x}$

$$f(x)=g'(x)=(\sin x)^{\ln x}\left[\cot x(\ln x)+\frac{\ln(\sin x)}{x}\right]$$

Hence $f\left(\frac{\pi}{2}\right)=g'\left(\frac{\pi}{2}\right)=1(0+0)=0$ (Ans.)

3. $$g'(0)=b=\lim_{x\to 0}\frac{x^2+x\tan x-x\tan 2x}{x(ax+\tan x-\tan 3x)}=\lim_{x\to 0}\frac{x+\tan x-\tan 2x}{ax+\tan x-\tan 3x}$$

$$=\lim_{x\to 0}\frac{x+\left(x+\frac{x^3}{3}+\frac{2}{15}x^5+......\infty\right)-\left(2x+\frac{8x^3}{3}+\frac{2}{15}.32x^5+......\infty\right)}{ax+\left(x+\frac{x^3}{3}+\frac{2}{15}x^5+......\infty\right)-\left(3x+\frac{27x^3}{3}+\frac{2}{15}.243x^5+......\infty\right)}$$

On simplifying a = 2, b= $\frac{7}{26}$.

4. We have $\lim\limits_{n\to\infty}\frac{n.3^n}{n(x-2)^n+n.3^{n+1}-3^n}=\frac{1}{3}$;

so $\lim\limits_{n\to\infty}\frac{1}{\left(\frac{x-2}{3}\right)^n+3-\frac{1}{n}}=\frac{1}{3}$

Clearly $-1<\frac{x-2}{3}<1\Rightarrow\quad -1<x<5$

$\therefore$ Possible integers in the range 'x' are 0,1,2,3,4 $\Rightarrow$ 5 integers.

5. For continuity of f at x = 0, we have

$$k=f(0)=\lim_{x\to 0}f(x)=\lim_{x\to 0}\frac{e^{\tan x}-e^x}{\tan x-x}+\lim_{x\to 0}\frac{\ln(\sec x+\tan x)-x}{\left(\frac{\tan x-x}{x^3}\right)x^3}$$

$$=\lim_{x\to 0}\frac{e^x\left(e^{\tan x-x}-1\right)}{\tan x-x}+3\lim_{x\to 0}\frac{\ln(\sec x-\tan x)-x}{x^3}$$

$=1+3\lim_{x\to 0}\frac{\sec x-1}{3x^2}$

(Using LH Rule)

$=1+\frac{1}{2}=\frac{3}{2}$

6. $\underset{x\to 0^+}{Lt} f(x)=\underset{h\to 0}{Lt}\frac{\sin^{-1}(1-h)}{\sqrt{2}(1-h)(1+h)}.\underset{h\to 0}{Lt}\frac{\cos^{-1}(1-h^2)}{h}=\frac{\pi}{2}.$

$\underset{x\to 0^-}{Lt} f(x)=\underset{h\to 0}{Lt}\frac{\cos^{-1}h(2-h).\sin^{-1}h}{\sqrt{2}(1-h)(2-h)}=\frac{\pi}{4\sqrt{2}}.$

7. Use sandwhich Theorem.

8. $:\because 2<\mathrm{f}(\mathrm{x})<\sqrt{26}$

$\therefore \frac{2}{c}<\frac{f(x)}{c}<\frac{\sqrt{26}}{c}$

since, $g(x)=\left[\frac{f(x)}{c}\right]$ is continuous $\forall x\in R$

$\Rightarrow g(x)=0$ for $c=6$

9. We have $\lim_{x\to 1^-} f(x)=\lim_{h\to 0} f(1-h)$

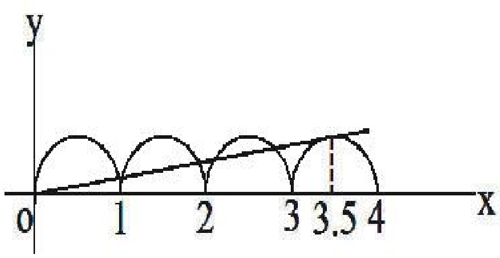
$=\lim_{h\to 0}\frac{\log(4+h^2)}{\log(1-4h)}=-\infty$

and

$\lim_{x\to 1^+} f(x)=\lim_{h\to 0} f(1+h)=\lim_{h\to 0}\frac{\log(4+h^2)}{\log(1+4h)}=\infty$

So, $f(1^-)$ and $f(1^+)$ do not exist.

10. $f(x)=\max\left\{\frac{x}{n},|\sin\pi x|\right\}$



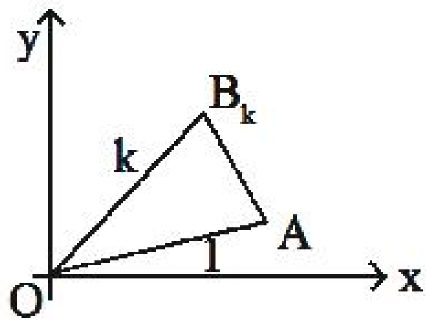
Thus, for the maximum points of non differentiability, graphs of $y=\frac{x}{7}$ and $y=|\sin\pi x|$ must intersect at maximum number of points which occurs when $n>3.5$. Hence, the least value of n is 4.

11. $OB_k=k$

$\angle AOB_k=\frac{k\pi}{2n}$

$S_k=\frac{1}{2}k\sin\frac{k\pi}{2n}$

(using $\Delta=\frac{1}{2}ab\sin\theta$)



$\therefore\ L=\frac{k}{2n^2}\sum_{k=1}^{\infty}\sin\frac{k\pi}{2n}=\frac{1}{2n}\sum_{k=1}^{\infty}\sin\frac{k\pi}{2n}=\frac{1}{2}\int_0^1 x\sin\frac{\pi x}{2}dx$

$=\frac{1}{2}\left[\underbrace{\frac{2}{\pi}x\cos\frac{\pi x}{2}\Big|_0^1}_{zero}\ \frac{2}{\pi}\int_0^1\cos\frac{\pi x}{2}dx\right]$

$=\frac{1}{2}\left[0+\frac{2}{\pi}.\frac{2}{\pi}\sin\frac{\pi x}{n}\Big|_0^1\right]=\frac{2}{\pi^2}$

12. We have $\sum_{n=2}^{\infty}\ln\left(1-\frac{1}{n^2}\right)$

$=\sum_{n=2}^{\infty}\ln\left(\frac{n^2-1}{n^2}\right)$

$=\sum_{n=2}^{\infty}\ln\left(\left(\frac{n-1}{n}\right)\left(\frac{n+1}{n}\right)\right)$

$$=\sum_{n=2}^{\infty}\left(\ln\left(\frac{n-1}{n}\right)+\ln\left(\frac{n+1}{n}\right)\right)$$

$$=\sum_{n=2}^{\infty}\left(\ln\left(\frac{n-1}{n}\right)-\ln\left(\frac{n}{n+1}\right)\right)$$

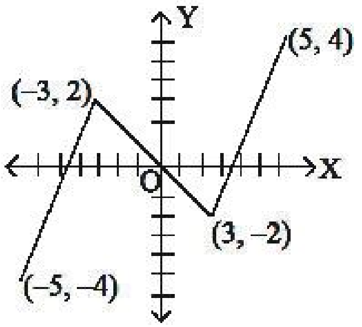
$$=\left(\ln\frac{1}{2}-\ln\frac{2}{3}\right)+\left(\ln\frac{2}{3}-\ln\frac{3}{4}\right)+..........$$

$$=\ln\frac{1}{2}-\underset{n\to\infty}{Lt}\left(\frac{n}{n+1}\right)=\ln\frac{1}{2}-\ln 1=-\ln 2$$

13. $f(x)+f(-x)=0\Rightarrow f(x)$ is an odd function. Since points $(-3,2)$ and $(5,4)$ lie on the curve

$\therefore$ $(3,-2)$ and $(-5,-4)$ will also lie on the curve.

For minimum number of roots, graph of continuous function $f(x)$ is as follows.



from the above two graphs of $f(x)$ it is clear that equation $f(x)=0$ has atleast three real roots.

14. Given $\left(1+\frac{1}{n}\right)^{n+x_n}=e$

taking log

$$(n+x_n)\ln\left(1+\frac{1}{n}\right)=1\Rightarrow n+x_n=\frac{1}{\ln\left(1+\frac{1}{n}\right)}$$

$$\Rightarrow x_n=\frac{1}{\ln\left(1+\frac{1}{n}\right)}-n \qquad .....(1)$$

let $\frac{n+1}{n}=u \Rightarrow nu=n+1 \Rightarrow n=\frac{1}{u-1}$

$$x_n=\underset{u\to1}{Lim}\left(\frac{1}{\ln u}-\frac{1}{u-1}\right)=\lim_{u\to1}\frac{(u-1)-\ln u}{(u-1)\ln u}$$

$$=\underset{u\to1}{Lim}\frac{1-\frac{1}{u}}{\frac{u-1}{u}+\ln u}=\underset{u\to1}{Lim}\frac{\frac{1}{u^2}}{\frac{1}{u^2}+\frac{1}{u}}=\frac{1}{2}$$

15. $\cos^3 x-\cos^2 x\to 0$ from LHS

$\sin^2 x-\sin^3 x\to 0$ from RHS

$\therefore$ the given line reduces to $4y=3x$

MULTI ANSWER TYPE QUESTIONS

16. $f(x)=\sum_{r=1}^{n}\left(x^{2r}+\frac{1}{x^{2r}}+2\right)=\sum_{r=1}^{n}x^{2r}+\sum_{r=1}^{n}\frac{1}{x^{2r}}+2n$

$$=\left(x^2+x^4+......+x^{2n}\right)+\left(\frac{1}{x^2}+\frac{1}{x^4}+.....+\frac{1}{x^{2n}}\right)+2n$$

$$=\frac{x^2\left(1-x^{2n}\right)}{\left(1-x^2\right)}+\frac{1}{x^2}\frac{\left(1-\frac{1}{x^{2n}}\right)}{1-\frac{1}{x^2}}+2n$$

$$f(x)=\frac{x^2\left(1-x^{2n}\right)}{1-x^2}+\frac{\left(1-x^{2n}\right)}{\left(1-x^2\right)x^{2n}}+2n$$

$$f(x)=\frac{\left(1-x^{2n}\right)}{1-x^2}\left(x^2+\frac{1}{x^{2n}}\right)+2n \quad x\neq\pm1$$

$$\therefore\ \left(f(x)-2n\right)\left(1-x^2\right)=\left(1-x^{2n}\right)\left(x^2+\frac{1}{x^{2n}}\right)$$

now consider

$g(x)=\lim_{n\to\infty}\left(\left(f(x)-2n\right)x^{-2n-2}\left(1-x^2\right)\right)$ for $x\neq\pm1$

$$=\lim_{n\to\infty}\left(1-x^{2n}\right)\left(x^2+\frac{1}{x^{2n}}\right).\frac{1}{x^{2n+2}};\ x\neq\pm1$$

$$=\lim_{n\to\infty}\frac{\left(1-x^{2n}\right)\left(x^{2n+2}+1\right)}{\left(x^{2n}\right)\left(x^{2n+2}\right)}$$

$$=\lim_{n\to\infty}\left(-1+\frac{1}{x^{2n}}\right)\left(1+\frac{1}{x^{2n+2}}\right)$$

now $g(x)=\begin{cases}-1 & if\ |x|>1\\ -\infty & if\ |x|<1\\ 0 & if\ x=\pm1\end{cases}$

now clearly g has non removable infinite type of discontinuity at x = 1 and –1
g is continuous at x = 2

17. $$\lim_{n\to\infty}\sum_{k=1}^{n}\frac{n}{n^2\left(a+\frac{k}{n}\right)\left(b+\frac{k}{n}\right)}$$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\frac{1}{\left(a+\frac{k}{n}\right)\left(b+\frac{k}{n}\right)}$$

$$=\int_0^1\frac{1}{(a+x)(b+x)}dx=\frac{1}{a-b}\int_0^1\frac{(a+x)-(b+x)}{(a+x)(b+x)}dx$$

$$=\frac{1}{a-b}\int_0^1\left(\frac{1}{b+x}-\frac{1}{a+x}\right)dx$$

$$=\frac{1}{a-b}\left[\ln(b+x)-\ln(a+x)\right]_0^1=\frac{1}{a-b}\left[\ln\frac{(b+x)}{(a+x)}\right]_0^1$$

$$=\frac{1}{a-b}\ln\frac{(b+1)a}{(a+1)b}$$

if a = b,

$$l=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{\left(a+\frac{k}{n}\right)^2}=\int_0^1\frac{dx}{(a+x)^2}=\left.\frac{1}{a+x}\right]_1^0$$

$$=\frac{1}{a}-\frac{1}{a+1}=\frac{1}{a(a+1)}$$

18. We have, $f(x)=\min\{1,\cos x,1-\sin x\}$

$\therefore$ f(x) can be rewritten as

$$f(x)=\begin{cases}\cos x, & -\frac{\pi}{2}\le x\le 0\\ 1-\sin x, & 0<x\le\frac{\pi}{2}\\ \cos x, & \frac{\pi}{2}<x\le\pi\end{cases}$$

$$\Rightarrow f'(x)=\begin{cases}-\sin x, & -\frac{\pi}{2}\le x\le 0\\ -\cos x, & 0<x\le\frac{\pi}{2}\\ -\sin x, & \frac{\pi}{2}<x\le\pi\end{cases}$$

$\because\ f'(0)=0$

hence, f(x) has local maxima at 0 and f(x) is not differentiable at x = 0.

19. : f(x) is discontinuous at $\frac{\pi}{2},\pi\frac{3\pi}{2},2\pi,.....$

and f(x) is periodic with period π

$$\int_0^{\pi/2}f(x)dx=\int_0^{\pi/4}f(x)dx+\int_{\pi/4}^{\pi/2}f(x)dx$$

$$=\int_0^{\pi/4}\tan x dx+\int_{\pi/4}^{\pi/2}\cot x dx$$

$$=\left[\ln\sec x\right]_0^{\pi/4}+\left[\ln\sin x\right]_{\pi/4}^{\pi/4}$$

$$=\left[\ln\sqrt{2}-0\right]+\left[0-\ln\left(\frac{1}{\sqrt{2}}\right)\right]$$

$$=2\ln\sqrt{2}$$

20. $$f(x)=\begin{cases}e^x & x<0\\ 2-e^x & 0\le x<\ln 2\\ e^x-2 & x\ge\ln 2\end{cases}$$

f is continuous $\forall x \in R$, but is not differentiable at $x = 0, \ln 2$

21. We have $\lim_{k \to 0^+} f(x) = \lim_{k \to 0^+} \frac{\tan^2\{x\}}{\left(x^2 - [x]^2\right)}$

$= \lim_{x \to 0^+} \frac{\tan^2 x}{x^2} = 1$ .....(1)

$\left[\because x \to 0^+; [x] = 0 = \{x\} = x\right]$

Also $\lim_{k \to 0^-} f(x) = \lim_{x \to 0^-} \sqrt{\{x\} \cot\{x\}} = \sqrt{\cot 1}$ .....(2)

$\left[\because x \to 0^-; [x] = -1 \Rightarrow \{x\} = x + 1 \Rightarrow \{x\} \to 1\right]$

Also, $\cot^{-1}\left(\lim_{x \to 0^-} f(x)\right)^2 = \cot^{-1}(\cot 1) = 1$

22. For maximum points of discontinuity of $f(x) = \operatorname{sgn}\left(x^2 - ax + 1\right)$, $x^2 - ax + 1 = 0$ must have two distinct roots, for which

$D = a^2 - 4 > 0$

$\Rightarrow a \in (-\infty, -2) \cup (2, \infty)$.

23. For continuity at x =1

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+}\left(x^2 \operatorname{sgn}[x] + \{x\}\right) = 1 + 0 = 1$

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-}\left(x^2 \operatorname{sgn}[x] + \{x\}\right) = 1\operatorname{sgn}(0) + 1 = 1$

Also, $f(1) = 1$

$\therefore$ L.H.L = R.H.L = $f(1)$.

Hence $f(x)$ is continuous at x =1.

Now for differentiability,

$f'(1^+) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$

$= \lim_{h \to 0} \frac{(1+h)^2 \operatorname{sgn}[1+h] + \{1+h\} - 1}{h}$

$= \lim_{h \to 0} \frac{(1+h)^2 + h - 1}{h}$

$= \lim_{h \to 0} \frac{h^2 + 3h}{h}$

= 3

and $f'(1^{-1}) = \lim_{h \to 0} \frac{f(1-h) - f(1)}{-h}$

$= \lim_{h \to 0} \frac{(1-h)^2 \operatorname{sgn}[1-h] + \{1-h\} - 1}{-h}$

$= \lim_{h \to 0} \frac{(1-h)^2 + 1 - h - 1}{-h}$

$= \lim_{h \to 0} \frac{h^2 - 3h}{-h}$

= 3

$f'(1^+) = f'(1^-)$

Hence, $f(x)$ is differentiable at x =1.

Now at x =2,

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-}\left(x^2 \operatorname{sgn}[x] + \{x\}\right) = 4 \times 0 + 1$

$\lim_{h \to 2^+} f(x) = \lim_{h \to 2^+}\left(\sin x + |x - 3|\right) = 1 + \sin 2$

Hence $LHL \neq RHL$

Hence, $f(x)$ is discontinuous at x =2 and then $f(x)$ is also non differentiable at x = 2.

24. (A): The expression $f(x) f(c)\ \forall\ x \in (c-h, c+h)$ where $h \to 0^+$ is equivalent to $\underset{x \to 0}{Lim} f(x) f(c)$ which equals to $\left(f(c)\right)^2$ because $f(x)$ is continuous.

$\therefore\ f(x) f(c) > 0\ \forall\ x \in (c-h,\ c+h)$ where $h \to 0^+$.

(B): We have

$I = \underset{n \to \infty}{Lim} \frac{1}{n} \ln\left[\left(1 + \frac{1}{n}\right)\left(1 + \frac{2}{n}\right)\ldots\left(1 + \frac{n}{n}\right)\right]$

$= \underset{n\to\infty}{Lim}\frac{1}{n}\ln\prod_{k=1}^{n}\left(1+\frac{k}{n}\right)$

$= \underset{n\to\infty}{Lim}\frac{1}{n}\sum_{k=1}^{n}\ln\left(1+\frac{k}{n}\right) = \int_1^2 \ln x dx = \left[x(\ln x-1)\right]_{x=1}^{x=2}$

$= -1+2\ln 2 \approx -0.4$

(C): Given $f(x)\ge 0 \Rightarrow \int_a^b f(x)dx \ge 0$

But given $\int_a^b f(x)dx = 0$, so this can be true only whe $f(x)=0$

(D): $\int_a^b f(x)dx = 0 \Rightarrow y = f(x)$ cuts X axis at least once

.

COMPREHENSION QUESTIONS

PASSAGE ::1 25 -27

$\because (gof)(x)$ is continuous $g(x)$ and $f(x)$ are also continuous

$\underset{x\to 0^-}{Lt} f(x) = f(0)$

$\underset{h\to 0}{Lt} (0-h)+a = 1$

$\Rightarrow a = 1$

$\underset{x\to 0^-}{Lt} g(x) = g(0)$

$\underset{h\to 0}{Lt} 0-h+1 = (0-1)^2 + b$

$\Rightarrow 1 = 1+b$

$\Rightarrow b = 0$

$(gof)(x) = g(f(x)) = \begin{cases} f(x)+1, & f(x)<0 \\ (f(x)-1)^2, & f(x)\ge 0 \end{cases}$

$= \begin{cases} x+2, & x<-1 \\ x^2, & -1\le x<1 \\ (x-2)^2, & x\ge 1 \end{cases}$

$\therefore (gof)(x)$ is even for all $x\in(-1,1)$

PASSAGE ::2 Q.No. 28 to 30

Conceptual

PASSAGE ::3 Q.No. 31 to 33

For $0\le x<\frac{\pi}{4}$, $g(x)=1+\tan x$

$x\in\left[0,\frac{\pi}{4}\right) \Rightarrow 1+\tan x\in[1,2)$

So $f(g(x)) = f(1+\tan x) = 1+\tan x+2$

and for $x\in\left[\frac{\pi}{4},\pi\right), g(x) = 3-\cot x$

$x\in\left[\frac{\pi}{4},\pi\right) \Rightarrow 3-\cot x\in[2,\infty)$

So $f(g(x)) = f(3-\cot x) = 6-(3-\cot x)$

Let

$h(x) = f(g(x)) = \begin{cases} 3+\tan x, & 0\le x<\frac{\pi}{4} \\ 3+\cot x, & \frac{\pi}{4}\le x<\pi \end{cases}$

Clearly, $f(g(x))$ is continuous in $[0,\pi)$

Now $h'\left(\frac{\pi^+}{4}\right) = \lim_{x\to\frac{\pi}{4^+}}\left(-\cos ec^2 x\right) = -2$

$h'\left(\frac{\pi^-}{4}\right) = \lim_{x\to\frac{\pi}{4^-}}\left(\sec^2 x\right) = 2$

So $f(g(x))$ is differentiable everywhere in $[0,\pi)$ other than at $x=\frac{\pi}{4}$.

$|f(g(x))| = \begin{cases} |3+\tan x|, & 0\le x<\frac{\pi}{4} \\ |3+\cot x|, & \frac{\pi}{4}\le x<\pi \end{cases}$

Which is non differentiable at $x=\frac{\pi}{4}$ and where $3+\cot x = 0$ or $x=\cot^{-1}(-3)$

For $x\in\left[0,\frac{\pi}{4}\right), 3+\tan x\in[3,4)$

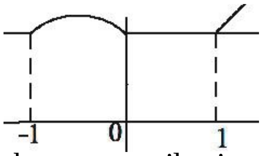
For $x\in\left[\frac{\pi}{4},\pi\right), 3+\cot x\in(-\infty,4]$

Hence, the range is $(-\infty,4]$.

PASSAGE ::4 Q.No. 34 to 36

$$f(t)=\begin{cases}3-t, & t\le 1\\ t+1, & t>1\end{cases}$$

$$g(x)=\begin{cases}3-\left(x^2+x+1\right) & x\in[-1,0)\\ 2 & x\in(-\infty,-1)\cup(0,1]\\ x+1 & x\in[1,\infty)\end{cases}$$



From graph we can easily give the answer

PASSAGES ::5 Q.No. 37 to 39

$a_{43}=a_{42}+d_4\Rightarrow d_4=\dfrac{1}{16}$ (common difference of 4th row)

$a_{41}=a_{42}-d_4=\dfrac{1}{16}$

$\therefore\ a_{41}=\dfrac{1}{16},\ a_{42}=\dfrac{2}{16},\ a_{43}=\dfrac{3}{16},\ a_{44}=\dfrac{4}{16},$ ...., $a_{4n}=\dfrac{n}{16}$

now all elements of 4th row are known

$S_n=\sum_{j=1}^{n}a_{4j}=\dfrac{n(n+1)}{2(16)}$;

$\lim_{n\to\infty}\dfrac{S_n}{n^2}=\dfrac{1}{32}$ (Ans: D)

$a_{24}r^2=a_{44}=\dfrac{4}{16}\Rightarrow r^2=\dfrac{1}{4}\Rightarrow r=\dfrac{1}{2}$ (common ratio for all G.P. is $\dfrac{1}{2}$)

note that all the lements of 4th row are known and common ratio $\dfrac{1}{2}$ is also known therefore $(m\times n)$ matrix is constructed.

$$A=\begin{bmatrix}\frac{1}{2} & \frac{2}{2} & \frac{3}{2} & \frac{4}{2} & \cdots & \frac{n}{2}\\ \frac{1}{2^2} & \frac{2}{2^2} & \frac{3}{2^2} & \frac{4}{2^2} & \cdots & \frac{n}{2^2}\\ \frac{1}{2^3} & \frac{2}{2^3} & \frac{3}{2^3} & \frac{4}{2^3} & \cdots & \frac{n}{2^3}\\ \frac{1}{2^4} & \frac{2}{2^4} & \frac{3}{2^4} & \frac{4}{2^4} & \cdots & \frac{n}{2^4}\\ \frac{1}{2^n} & \frac{2}{2^n} & \frac{3}{2^n} & \frac{4}{2^n} & \cdots & \frac{n}{2^n}\end{bmatrix}$$

Working: $a_{41}=\dfrac{1}{2^4}$, $a_{42}=\dfrac{2}{2^4}$

|||ly $a_{4i}=\dfrac{i}{2^4}$

now $\left.\begin{matrix}a_{11}\\a_{21}\\a_{31}\\a_{41}\end{matrix}\right\}$ in G.P. $a_{41}=a_{11}.r^3$ $\Rightarrow \dfrac{1}{16}=a_{11}.\dfrac{1}{8}$ $\Rightarrow a_{11}=\dfrac{1}{2}$

again $a_{42}=\dfrac{2}{2^4}$

$\left.\begin{matrix}a_{12}\\a_{22}\\a_{33}\\a_{42}\end{matrix}\right\}$ in G.P. $a_{42}=a_{12}.r^3$ $\Rightarrow \dfrac{2}{16}=a_{12}.\dfrac{1}{8}$ $\Rightarrow a_{12}=\dfrac{1}{2^2}$

$$\sum_{i=1}^{n}d_i=\sum_{i=1}^{n}\frac{1}{2^i}=\frac{\frac{1}{2}\left(1-\frac{1}{2^n}\right)}{\left(1-\frac{1}{2}\right)}=1-\frac{1}{2^n}$$

$$\sum_{i=1}^{n}a_{ii}=S=\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+\frac{4}{2^4}+.....+\frac{n}{2^n}$$

$$\frac{1}{2}S=\frac{1}{2^2}+\frac{2}{2^3}+\frac{3}{2^4}+.....+\frac{n-1}{2^n}+\frac{n}{2^{n+1}}$$

$$\frac{1}{2}S=\left(\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+.....+\frac{1}{2^n}\right)-\frac{n}{2^{n+1}}$$

$$S=2\left(1-\frac{1}{2^n}\right)-\frac{n}{2^n}$$

$$\lim_{n\to\infty}\sum_{i=1}^{n}a_{ii}=\lim_{n\to\infty}\left(2-\frac{2}{2^n}-\frac{n}{2^n}\right)$$

$$=2-0-0\Rightarrow \lim_{n\to\infty}\sum_{i=1}^{n}a_{ii}=2$$

PASSAGE: 6 q.no. 40 to 42

$$g(x)=\begin{matrix}f(x) & if & -2\le x\le -1\\ -1 & if & -1\le x<0\\ 0 & if & -\le x<1\\ f(x) & if & 1\le x\le 3\end{matrix}$$

Graph of $y=f(x)$ is

Graph of $y=g(x)$ is

PASSAGE: 7 q.no. 43 to 45

We have $f(x)=\begin{cases}3x, x\geq 0\\ x, \quad x<0\end{cases}$ and

$$g(x)=\begin{cases}\dfrac{x}{3}, x\geq 0\\ x, \quad x<0\end{cases}$$

Clearly f and g are inverse of each other

Now, $h(x)=f(g(x))=\begin{cases}3\left(\dfrac{x}{3}\right)=x,\ x\geq 0\\ x, \qquad\qquad x<0\end{cases}$

(i) As $h(x)=x\forall x\in R$

$\Rightarrow\ k(x)=1+\dfrac{1}{\pi}\left(\cos^{-1}x+cot^{-1}x\right)$

Domain of $k(x)=[-1,1]$ and $k(x)$ is decreasing function on $[-1,1]$

As $k(x)$ is continuous function on $[-1,1]$

Now,

$$k_{\min}(x=1)=1+\frac{1}{\pi}\left(\cos^{-1}1+\cot^{-1}1\right)$$

$$=1+\frac{1}{\pi}\left(0+\frac{\pi}{4}\right)=1+\frac{1}{4}=\frac{5}{4}$$

$$k_{\max}(x=-1)=1+\frac{1}{\pi}\left(\cos^{-1}(-1)+\cot^{-1}(-1)\right)$$

$$=1+\frac{1}{\pi}\left(\pi+\frac{3\pi}{4}\right)=1+\frac{7}{4}=\frac{11}{4}$$

$\Rightarrow$ Range of $k(x)=\left[\dfrac{5}{4},\dfrac{11}{4}\right]$

(ii) We have

$$f(x)-g(x)=(2x+|x|)-\frac{1}{3}(2x-|x|)$$

$$=\frac{4x}{3}+\frac{4}{3}|x|=\begin{cases}\dfrac{8}{3}x;\ x\geq 0\\ 0\ ;\ x<0\end{cases}$$

$\therefore$ For domain of ducntion,

$0\leq\dfrac{8x}{3}\leq 1\Rightarrow \quad 0\leq x\leq\dfrac{3}{8}\ \Rightarrow$

Domain of $l(x)=\left(-\infty,\dfrac{3}{8}\right]$

Note : Range of function $l(x)=\left[0,\dfrac{\pi}{2}\right]$

(iii) As f and g are ianverse of each other, so

$$T0x=f\left(g\left(f(x)\right)\right)+g\left(f\left(g(x)\right)\right)$$

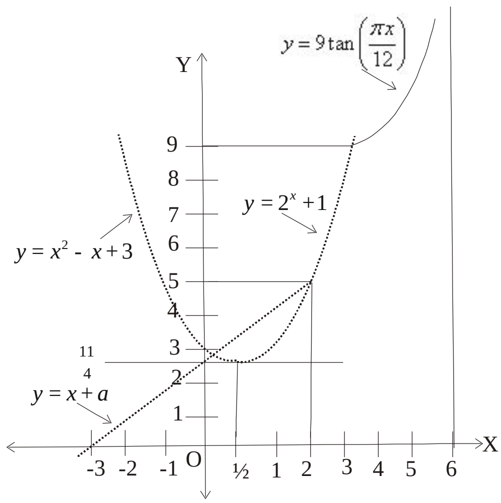
$$=f(x)+g(x)=(2x+|x|)+\frac{1}{3}(2x-|x|)$$

$$\Rightarrow\ T(x)=\begin{cases}\dfrac{10x}{3}, x\geq 0\\ 2x, \quad x<0\end{cases}$$

Clearly, $T(x)$ is continuous but non-deriable at x = 0

PASSAGES:: 8 Q.NO. 46 to 48

$$f(x)=\begin{cases}x^2-x+3,\ if\ \ x\in Q\ \ in\ (-\infty,3)\\ x+a, \qquad if\ \ x\notin Q\ in\ (-\infty,2)\\ 2^x+1, \qquad if\ \ x\notin Q\ in\ (2,3)\\ 9\tan\left(\dfrac{\pi x}{12}\right), if\ x\geq 3\end{cases}$$



$f(2^+)=2^2+1=5$

through irrational

$f(2^-)=2+a$

through rational

$f(2)=4-2+3=5$

Hence for continuity at x = 2

$2+a=5\Rightarrow a=3$

for $x \geq 3$

$f(x) = 9\tan\frac{\pi x}{12}$

$\Rightarrow f(3^+) = 9$; $f(3^-) = 2^3 + 1 = 9$

$\Rightarrow$ f is continuis at x = 3 obvious not derivable

$f'(x) = 9\sec^2\left(\frac{\pi x}{12}\right)\frac{\pi}{12}$

$f'(4) = 9\sec^2\left(\frac{\pi}{3}\right)\frac{\pi}{12} = 3\pi$

MATRIX MATCHING QUESTIONS

49. (A) $f(x) = \begin{cases} -x, x < 0 \\ x, x \geq 0 \end{cases}$

continuous but not differentiable at $x = 0$

(B) $f(x) = x^n|x|$

$\Rightarrow LHD = RHD = 0$ at $x = 0$

(C) $LHL = RHL = f(0) = 0$ but LHD and RHD are not finite

(D) $LHL = 0, RHL = \underset{x\to 0}{Lt}\frac{e^{1/x}}{1/x}$

$= \underset{x\to 0}{Lt}\frac{e^{1/x}(-1/x^2)}{(-1/x^2)} = \underset{x\to 0}{Lt}\, e^{1/x} \to \infty$

50. A) Thegiven function is clearly continuous at all points except possibly at $x = \pm 1$.

As $f(x)$ is an even function, so we need to check its continuity only at x =1.

$\Rightarrow \lim_{x\to 1^-}(ax^2 + b) = \lim_{x\to 1^+}\frac{1}{|x|} \Rightarrow a = 1 = b = 1$

Clearly, $f(x)$ is differentiable for all x, except possibly at $x = \pm 1$. As $f(x)$ is an even function, so we need to check its differentiability at x=1 onlyu.

$\lim_{x\to 1^-}\frac{f(x) - f(1)}{x-1} = \lim_{x\to 1^+}\frac{f(x) - f(1)}{x-1}$

$\Rightarrow \lim_{x\to 1}\frac{ax^2 + b - 1}{x-1} = \lim_{x\to 1}\frac{ax^2 + b - 1}{x-1} = \lim_{x\to 1^+}\frac{\frac{1}{|x|} - 1}{x-1}$

$\Rightarrow \lim_{x\to 1}\frac{ax^2 - a}{x-1} = \lim_{x\to 1}\frac{-1}{x} \Rightarrow 2a = -1 \Rightarrow a = \frac{1}{2\frac{1}{2}}$

Putting $a = -\frac{1}{2}$ in (1) we get

$b = \frac{3}{2} \Rightarrow |k| = 1 \Rightarrow k = \pm 1$

B) $f(x) = \text{sgn}(x^2 - ax + 1)$ is discontinuous then $x^2 - ax + 1 = 0$ must have only one real root. Hence $a = \pm 2$.

C) $f(x) = [2 + 3|n|\sin x], n \in N$ has exactly 10 points of discontinuity in $x \in (0, \pi)$. The required numnber of points are

$1 + 2(3|n| - 1)$

$= 6|n| - 1 = 11 \Rightarrow n = \pm 2$

D) If $f(x) = ||x| - 2| + a|$ has exactly three points of non differentiability.

$f(x)$ is non-differentiable at x=0, $|x| - 2 = 0$ or $x = 0, \pm 2$.

Hence, the value of a must be positive, as negative value of a allows $||x| - 2| + a = 0$ to have real roots, which gives to more points of non differentiability.

51. $g(x)$ can be defined as

$g(x)\begin{cases} f(x), -2 \leq x < -1 \\ -1 \quad , -1 \leq x < 0 \\ f(x), \; 2 \leq x \leq 3 \end{cases}$

or $g(x)\begin{cases} x^2 + 2x, \; -2 \leq x < -1 \\ -1, \quad -1 \leq x < 0 \\ x^2 - 2x, \; 2 \leq x \leq 3 \end{cases}$

Clearly $g(x)$ is discontinuous at x = 0

$g(x)$ is not differentiable at x =0, 2

Ab solute maximum value of $g(x) = f(3) = 3$.

52. (A): RHL

$$\lim_{h\to 0}\frac{\left(\frac{\pi}{2}-\sin^{-1}\left(1-h^2\right)\right)\sin^{-1}(1-h)}{\sqrt{2}\left(h-h^3\right)}$$

$$\lim_{h\to 0}\frac{\cos^{-1}\left(1-h^2\right)\sin^{-1}(1-h)}{\sqrt{2}h\left(1-(h)\right)^2}$$

$$\lim_{h\to 0}\frac{\sin^{-1}\sqrt{2h^2-h^2}\,\sin^{-1}(1-h)}{\sqrt{2}h\left(1-(h)\right)^2}$$

$$\lim_{h\to 0}\frac{\sin^{-1}\sqrt{2h^2-h^2}\,\sin^{-1}(1-h)}{\sqrt{2}h\left(1-(h)\right)^2}$$

$$\Rightarrow k=\frac{\pi}{2}$$

LHL.

$$\lim_{h\to 0}A\frac{\sin^{-1}\left(1-(1-h)\right)\cos^{-1}\left(1-(1-h)\right)}{\sqrt{2(1-h)}.h}$$

$$\lim_{h\to 0}A\frac{\sin^{-1}h\cos^{-1}h}{\sqrt{2(1-h)}.h}=A\frac{\pi}{2\sqrt{2}}$$

$$\Rightarrow A\frac{\pi}{2\sqrt{2}}=\frac{\pi}{2}\Rightarrow A=\sqrt{2}$$

$$\therefore\ \sin^2 k+\cos^2\left(\frac{A\pi}{\sqrt{2}}\right)=1+1=2$$

(B): $\left[2+5|n|\sin x\right]=2+\left[5|n|sinx\right]$

$y=5|n|\sin x$

No. of points of non diff.

$=2\left(5|n|-1\right)+1$

$=10|n|-1$

$10|n|-1=19$

$10|n|-1=20$

$|n|=2$

$n=\pm 2$

(C):

RHD : $\lim_{h\to 0}h\dfrac{\left(\dfrac{\left(\frac{3}{4}\right)^{\frac{1}{h}}-\left(\frac{3}{4}\right)^{-\frac{1}{h}}}{\left(\frac{3}{4}\right)^{\frac{1}{h}}+\left(\frac{3}{4}\right)^{-\frac{1}{h}}}\right)}{h}=-1$

LHD $=\lim_{h\to 0}(-h)\dfrac{\dfrac{\left(\frac{3}{4}\right)^{-\frac{1}{h}}-\left(\frac{3}{4}\right)^{\frac{1}{h}}}{\left(\frac{3}{4}\right)^{-\frac{1}{h}}-\left(\frac{3}{4}\right)^{\frac{1}{h}}}}{-h}=1$

$$\therefore\ f'\left(0^-\right)-f'\left(0^+\right)=2=P$$

Now $\lim_{x\to 2^-}\dfrac{\left(exp(x+2)\ln 4\right)^{\frac{[x+1]}{4}}-16}{4^x-16}=\dfrac{1}{2}$

INTEGER QUESTIONS

53. $f(x)=\begin{cases}e^{-x},\ if\ \ x\ge 0\\ e^{x},\ if\ \ x>0\end{cases};$

$$f'\left(0^-\right)=\lim_{h\to 0}\frac{f(0-h)-f(0)}{-h}=\lim_{h\to 0}\frac{e^{-h}-1}{-h}=1$$

ence $\lim_{x\to 0}\dfrac{\left[1-\cos\left(1-\cos\frac{x}{2}\right)\right]}{\left(1-\cos\frac{x}{2}\right)^2}\dfrac{\left(1-\cos\frac{x}{2}\right)}{2^m.x^n}=1$

Using $\lim_{x\to 0}\dfrac{1-\cos\theta}{\theta^2}=\dfrac{1}{2}$

$$\lim_{x\to 0}\frac{\left(1-\cos\frac{x}{2}\right)}{2^m.x^n}=2$$

$$\lim_{x\to 0}\frac{4\sin^4\frac{x}{4}}{2^m.x^n}=2$$

$$\lim_{x\to 0}\frac{4x^4}{4^4 2^m.x^n}=2$$

$\therefore$ for limit to exist n = 4

$$\frac{2}{2^{8+m}}=1$$

$$2^{8+m}=2$$

$\therefore$ $m=-7 \Rightarrow n=4$ and $m=-7$

Hence $n^2+m=4^2+(-7)=16-7=9$.

54. i) Continuity should be checked at the end-points of intervals of each definition, i.e., x =1, 3, 3.5,....

ii) For $\{x^2\}$, continuity should be chedk when $x^2=10,11,12$ or $x\sqrt{10},\sqrt{11},\sqrt{12}$, $\{x^2\}$ is discontinuous for those values of x, where x^2 is an integer.

[Note: Here x^2 is monotonic for the tgiven domain.]

iii) For $\operatorname{sgn}(x-2)$, continuity should be checked when $x-2=0$ or $x=2$

iv) For $[\log_e x]$, continuity should be checked when $\log_e x=1$ or $x=e(\in[1,3])$.

Hence, the overall continuity must be checked at $x=1,2,e,3,\sqrt{10},\sqrt{11},\sqrt{12},3.5$.

Further, $f(1)=0$ and

$$\lim_{x\to 1^+} f(x)=\lim_{x\to 1^+}\operatorname{sgn}(x-2)\times[\log_e x]=0.$$

Hence $f(x)$ is continuous at x =1,

$$\lim_{x\to 2^-} f(x)=\lim_{x\to 2^+}\operatorname{sgn}(x-2)\times[\log_e x]=(-1)\times 0=0$$

$$\lim_{x\to 2^+} f(x)=\lim_{x\to 2^-}\operatorname{sgn}(x-2)\times[\log_e x]=(1)\times 0=0$$

Hence, $f(x)$ is continuous at x =2,

$$\lim_{x\to 3^-} f(x)=\lim_{x\to 3^-}\operatorname{sgn}(x-2)\times[\log_e x]=1$$

$$\Rightarrow \lim_{x\to 3^+} f(x)=\lim_{x\to 3^+}\{x^2\}=0$$

Hence, $f(x)$ is discontinuous at x =3.

Also $\{x^2\}$ is discontinuous at

$x=\sqrt{10},\sqrt{11},\sqrt{12}$,

Therefore,

$$\lim_{x\to 35^-} f(x)=\lim_{x\to 3.5^-}\{x^2\}=0.25=f(3.5).$$

Hence, $f(x)$ is discontinuous at

$x=3,\sqrt{10},\sqrt{11},\sqrt{12}$.

55. Putting $x=1+h$, we get

$$f(a)=\lim_{h\to 0}\frac{(1+h)^a-a(1+h)+a-1}{h^2}$$

$$=\lim_{h\to 0}\frac{\left(1+ah\frac{a(a-1)}{2!}h^2+......\right)-a-ah+a-1}{h}$$

$$=\frac{a(a-1)}{2}$$

Therefore $f(4)=6$.

56 & 57. Conceptual

58. Let

$$S=\sum_{r=1}^{\infty}\frac{r+2}{2^{r+1}.r.(r+1)}=\sum_{r=1}^{\infty}\frac{2(r+1)-r}{2^{r+1}.r.(r+1)}$$

$$=\sum_{r=1}^{\infty}\frac{1}{2^{r+1}}\left(\frac{2}{r}-\frac{1}{r+1}\right)=\sum_{r=1}^{\infty}\left(\frac{1}{2^r.r}-\frac{1}{2^{r+1}(r+1)}\right)$$

$$=\lim_{n\to\infty}\left[\left(\frac{1}{2^1.1}-\frac{1}{2^2.2}\right)+\left(\frac{1}{2^2.2}-\frac{1}{2^3.3}\right)\right]+\left(\frac{1}{2^3.3}-\frac{1}{2^4.4}\right)$$

$$+.....+\left(\frac{1}{2^n.n}-\frac{1}{2^{n+1}.(n+1)}\right)$$

$$=\lim_{n\to\infty}\left(\frac{1}{2}-\frac{1}{2^{n+1}.(n+1)}\right)$$

$\therefore$ $S=\frac{1}{2}$ Hence $S^{-1}=2$

59. Let given limit = L, then

$$L=\lim_{n\to\infty}\left(\frac{1}{2n+1}+\frac{1}{2n+2}+\frac{1}{2n+3}+.....+\frac{1}{4n}\right)-$$

$$\lim_{n\to\infty}\left(\frac{1}{2n+2}+\frac{1}{2n+4}+\frac{1}{2n+6}+.....+\frac{1}{4n}\right)$$

$$=\lim_{n\to\infty}\left[\frac{1}{n}\sum_{r=1}^{2n}\frac{n}{2n+r}-\frac{1}{n}\sum_{r=1}^{n}\frac{n}{2n+2r}\right]$$

$$=\lim_{n\to\infty}\left[\frac{1}{n}\sum_{r=1}^{2n}\frac{n}{2+\frac{r}{n}}-\frac{1}{n}\sum_{r=1}^{n}\frac{n}{2+2\left(\frac{r}{n}\right)}\right]$$

$$=\int_0^2\frac{1}{2+x}dx-\int_0^1\frac{1}{2+2x}dx$$

$$=\left[\ln(2+x)\right]_0^2-\frac{1}{2}\left[\ln(1+x)\right]_0^1$$

$$=\ln 4-\ln 2-\frac{1}{2}\ln 2=\left(2-\frac{3}{2}\right)\ln 2$$

$$=\frac{1}{2}\ln 2=\frac{a}{b}\ln c$$

$\therefore$ Least sum $a+b+c=1+2+2=5$.

60. We have

$$I_n=2\int_0^1 x\left(1+\frac{x^2}{2}+\frac{x^4}{4}+\frac{x^6}{6}+...+\frac{x^{2n}}{2n}\right)dx$$

$$\left(\int_{-1}^{1}(odd)\,dx=0\right)$$

$$=2\left[\frac{x^2}{1.2}+\frac{x^4}{2.4}+\frac{x^6}{4.6}+.....+\frac{x^{2n+2}}{2n(2n+2)}\right]_0^1$$

$$=2\left[\frac{1}{1.2}+\frac{1}{2.4}+\frac{1}{4.6}+.....+\frac{1}{2n(2n+2)}\right]$$

$$=1+\frac{1}{2}\left[\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+......+\left(\frac{1}{n}-\frac{1}{n+1}\right)\right]$$

Hence $\lim_{n\to\infty}\left[1+\frac{1}{2}\left(1-\frac{1}{n+1}\right)\right]=\frac{3}{2}$

P = 3; q = 2

61. It is obvious n is even, then

$$\lim_{n\to\infty}\left(2^{1+3+5+....+n/2\,terms}\times 3^{2+4+6+....+n/2\,terms}\right)^{\frac{1}{(n^2+1)}}$$

$$=\lim_{n\to\infty}\left(2^{\frac{n^2}{4}}\times 3^{\frac{n(n+2)}{4}}\right)^{\frac{1}{(n^2+1)}}$$

$$=\lim_{n\to\infty}2^{\frac{n^2}{4(n^2+1)}}\times 3^{\frac{n(n+2)}{4(n^2+1)}}$$

$$=2^{\lim_{n\to\infty}\frac{1}{4\left(1+\frac{1}{n^2}\right)}}\times 3^{\lim_{n\to\infty}\frac{\left(1+\frac{2}{n}\right)}{4\left(1+\frac{1}{n^2}\right)}}$$

$$=2^{\frac{1}{4}}3^{\frac{1}{4}}=(6)^{\frac{1}{4}}$$

62. $$\lim_{x\to 0}\frac{\cos^2 x-\cos x-e^x\cos x+e^x-\frac{x^3}{2}}{x^n}$$

$$=\lim_{x\to 0}\frac{(\cos x-1)(\cos x-e^x)-\frac{x^3}{2}}{x^n}$$

$$=\lim_{x\to 0}\frac{\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+......-1\right)}{x^n}$$

$$=\frac{\left[\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-.....\right)-\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}-.....\right)\right]-\frac{x^3}{2}}{x^n}$$

$$=\lim_{x\to 0}\frac{\left(-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^5}{6!}+.....\right)\left[\left(-x-x^2-\frac{x^3}{3!}-\frac{2x^5}{5!}-...\right)\right]-\frac{x^3}{2}}{x^n}$$

$$=\lim_{x\to 0}\frac{\left(\frac{x^3}{2}+\frac{x^4}{2}+\frac{x^5}{12}-\frac{x^5}{24}+.....\right)-\frac{x^3}{2}}{x^n}$$

= Non zero if $n = 4$

63. $\lim_{x\to 0}\frac{\sin x^4 - x^4\cos x^4 + x^{20}}{x^4\left(e^{2x^4}-1-2x^4\right)}$

$=\lim_{t\to 0}\frac{\sin t - t\cos t + t^5}{t\left(e^{2t}-1-2t\right)}$

$=\lim_{t\to 0}\frac{t-\frac{t^3}{3!}+\frac{t^5}{5!}....-t\left(1-\frac{t^2}{2!}+\frac{t^4}{4!}....\right)+t^5}{t\left(1+2t+\frac{4t^2}{2!}+\frac{8t^3}{3!}+\frac{16t^4}{4!}+....-1-2t\right)}$

$=\lim_{t\to 0}\frac{-\frac{t^3}{6}+\frac{t^3}{2}+\frac{t^5}{5!}-\frac{t^5}{4!}+....+t^5}{2t^3+\frac{8t^4}{3!}+....}$

$=\frac{-\frac{1}{6}+\frac{1}{2}}{2}=-\frac{-1+3}{12}=\frac{1}{6}$

64. $\lim_{x\to 0}\sin\left(\frac{\pi\left(1-\cos^m x\right)}{x^n}\right)$

$=\sin\left(\lim_{x\to 0}\frac{\pi\left(1-\cos^m x\right)}{x^n}\right)$

$=\sin\left(\lim_{x\to 0}2\pi m\frac{\sin^2 x/2}{x^n}\right)$

$\Rightarrow m\in N$ and $n=1$ or 2

65. $L=e^{\lim_{x\to\infty}\frac{\ln\left(e^{2x}+e^x+x\right)}{x}}$
using L Hospital's rule

$L=e^{\lim_{x\to\infty}\frac{2e^{2x}+e^x+1}{e^{2x}+e^x+x}}$

$=e^{\lim_{x\to\infty}\frac{2+e^{-x}+e^{-2x}}{1+e^{-x}+xe^{-2x}}}=e^2$

66. $\sin^{-1}x$ is a monotonically increasing function. Hence, $f(x)=\left[\sin^{-1}x\right]$ is discontinuous, where $\sin^{-1}x$ is an integer.

$\Rightarrow \sin^{-1}x=-1,0,1$ or $x=-\sin 1, 0, \sin 1$

$\frac{2}{1+x^2}, x\geq 0$, is a monotonically decreasing function.

Hence, $f(x)=\left[\frac{2}{1+x^2}\right], x\geq 0$ is discontinuous when $\frac{2}{1+x^2}$ is an integer.

$\Rightarrow \frac{2}{1+x^2}=1,2$

$\Rightarrow x-1,0$

67. $\sin(x-2)$ and $\cos(x-2)$ are continuous for all x. Sicne $\left[x^3\right]$ is not continuous at integral values of x^3, $f(x)$ is continous in $[0,2]$ if $\left[\frac{(x-2)^3}{a}\right]=0$, $\forall x\in[0,2]$.

Now, $(x-2)^3\in[0,8]$ for $x\in[4,6]$

$\Rightarrow a>8$ for $\left[\frac{(x-2)^3}{a}\right]=0$

68. $f(x)=\begin{cases}x-1, & x\,is\,rational\\ x^2-x-2, & x\,is\,irrational\end{cases}$ is continuous when $x-1=x^2-x-2$

or $x^2-2x-1=0$ or $x=\frac{2\pm\sqrt{8}}{2}$.

Hence, $f(x)$ is continuous at two points

$\Rightarrow a=2\cdot$

$f(x)=\text{sgn}\left(x^3-3x+1\right)$ is discontinuous when $x^3-3x+1=0\cdot$

Now, $y=x^3-3x+1$ has three distinct roots as $\frac{dy}{dx}=3x^2-3=0\Rightarrow x\pm 1$

Also $f(1)=1-3+1=-1$ and $f(-1)=-1+3+1=3$. Hence, the graph of $y=x^3-3x+1$ is

Hence, $f(x)=\text{sgn}\left(x^3-3x+1\right)$ is discontinuous at three points $\Rightarrow b=3$.

$f(x)=(\log x)\left|x^2-4x+3\right|+2(x-2)^{1/3}$

$=(\log x)|x-1||x-3|+2(x-2)^{1/3}$

Which is non-differentiable at $x=3$ and $x=2$ as at $x=3$, $f(x)$ has a sharp turn and at $x=2$, $f(x)$ have a vertical tangent. At $x=1$, $f(x)$ is differentiable. So $c=2$ and $d=1$

69. $$f(x)=\lim_{n\to\infty}\frac{(x^2)^n-1}{(x^2)^n+1}$$

$$=\lim_{n\to\infty}\frac{1-\frac{1}{(x^2)^n}}{1+\frac{1}{(x^2)^n}}$$

$$=\begin{cases}-1, & 0\le x^2<1\\ 0, & x^2=1\\ 1, & x^2>1\end{cases}=\begin{cases}1, & x<-1\\ 0, & x=-1\\ -1, & -1<x<1\\ 0, & x=1\\ 1, & x>1\end{cases}$$

Thus $f(x)$ is discontinuous at $x=\pm1$.69.

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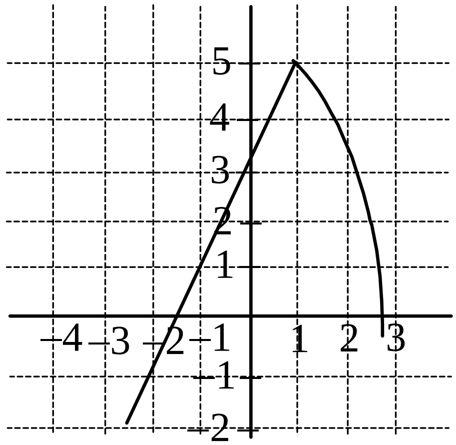
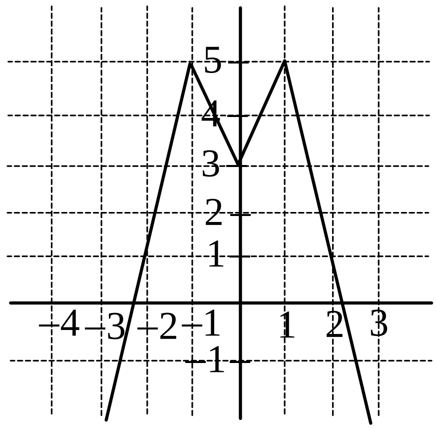
Thus $f(x)$ is discontinuous at $x=\pm1$.

70.

Graph of $y=f(x)$



Graph of $y=f(|x|)$



Graph of $y=|f(|x|)|$